How nonlinear systems turn out to quasi-linear: Hommage to Ivo Marek and M-matrices

by

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Nonlinear cooperative systems possessing certain conservation laws, frequently arising in biology and chemistry, were studied by Erich Bohl and Ivo Marek in different papers. However, the impact of these papers does not correspond to their originality and applicability.

The lecture will present the basic idea behind the Bohl-Marek formulation of the nonlinear system of ODEs; i.e., for a class of mathematical models with conservation laws, it will be shown how to transform systems of nonlinear ODEs into a set of smaller, quasi-linear subsystems in the form of $\dot{x}_j(t) = M_j(x_k(t))\, x_j(t)$, $j \neq k$. Moreover, $M_j(x_k(t))$, $j = 1, \ldots L$, are negative M-matrices whose entries depend on the variables x_k of the other subsystems involved, thus, the nonlinearity of the whole system is created via this inter-dependence: $\dot{x}(t) = A\, x(t)$, $A = \mathrm{diag}\,(M_1, \ldots, M_L)$.

While Erich Bohl and Ivo Marek used this method to prove the existence and uniqueness results, here, on a simple case study, namely Michaelis-Menten enzymatic reaction network, we show computational advantages when compared with solving the original system of nonlinear ODEs.

Only certain basic knowledge of mathematics is assumed; rather than a detailed overview of M-matrix algebra, the lecturer will surf 2-3 waves of applied mathematics and chemistry (smoothly accessible to undergraduate students).